

# Credit line exposure at default modeling using Bayesian mixed effect quantile regression

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# Agenda

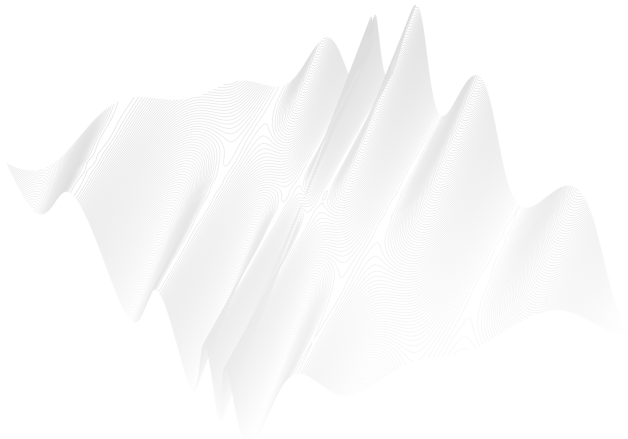
Introduction

Data

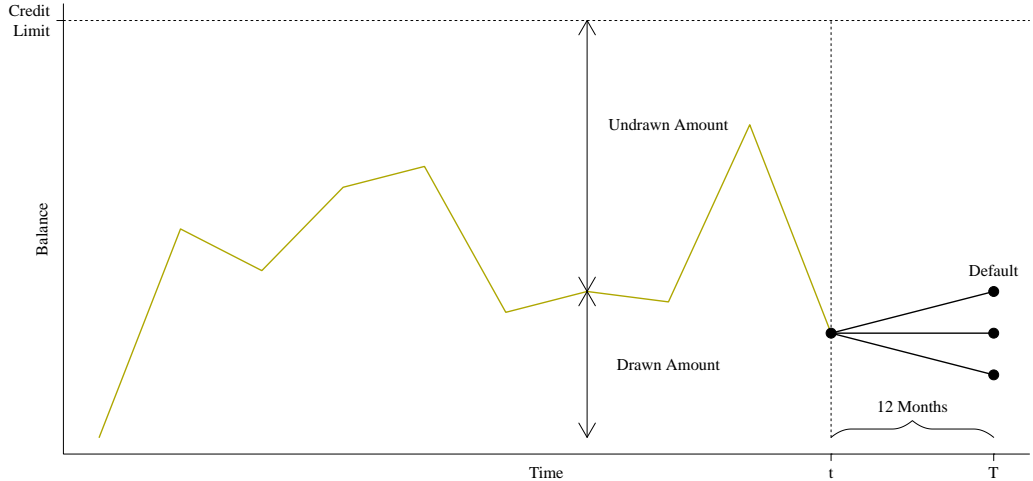
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Conclusion



# Exposure of credit lines



## Modeling approach

Credit conversion factors are used to model the additional draw down based on balance and limit at time  $t$ :

$$AUF = \begin{cases} \frac{EAD_T - Balance_t}{Limit_t} & \text{for } Limit_t \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

The Exposure at Default can be estimated as:

$$EAD_T = Balance_t + AUF \cdot Limit_t$$

## Advantages of conversion factors over direct estimation

- ▶ The Basel Accord motivates financial institutions to predict an additional drawdown (Basel Committee on Banking Supervision 2017)
- ▶ Evaluation of different drivers for low and high additional drawdowns can enhance line management

## Direct estimation

- ▶ Leow and Crook (2016), Tong et al. (2016) and Thackham and Ma (2018) estimate EAD using mixture models
- ▶ Hon and Bellotti (2016) uses a random effects panel model

## Conversion factors

- ▶ Araten and Jacobs Jr (2001), Moral (2006), Qi (2009), Barakova and Parthasarathy (2013) and Zhao et al. (2014) employ OLS regression
- ▶ Jacobs Jr (2010) uses a beta regression

## Contribution

We propose a flexible approach to model the **entire conditional distribution** of conversion factors and incorporate the **time varying** nature of credit lines

1. The empirical results show a **strong varying** impact of covariates over the conditional distribution
2. We show that the distributional fit of the most common method is **rather poor**, especially in downturn periods and in an out-of-time analysis
3. The analysis for European credit lines reveals, that **the tails** of the conditional distribution are driven by random effects, rather than observable covariates
4. The paper suggests that the macroeconomic environment is most important for **less drawn lines**

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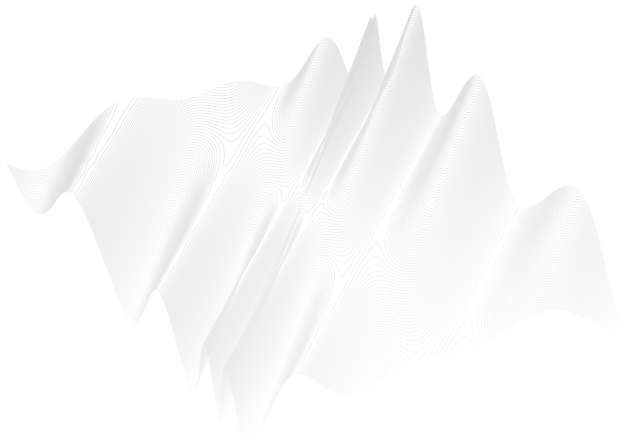
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### Subsample — Global Credit Data (GCD) database

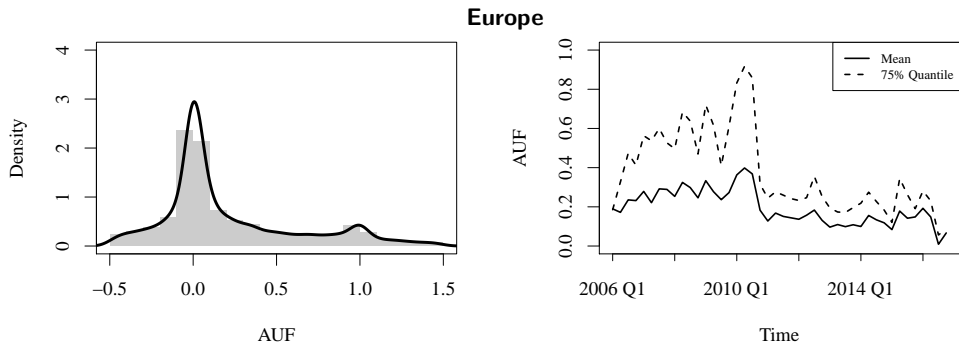
- ▶ European sample includes 14,382 individual corporate credit lines  
Sample consists of the most common European countries in data base
- ▶ US American sample includes 4,432 individual corporate credit lines
- ▶ 2006 to 2018 (2016 -2018 as out-of-time sample)

### Independent variables

1.  $\log(\text{Limit})$
2.  $\Delta \text{ GDP}$
3. Line age
4. Utilization
5. Seniority      pari-passu – super senior — non senior
6. Industry      FIRE – Agriculture — Mining — Construction — Manufacturing —  
Transportation — Wholesale — Service — Other
7. Facility      medium term revolver – short term revolver — overdraft



## Data — Distribution & time variation



# Agenda

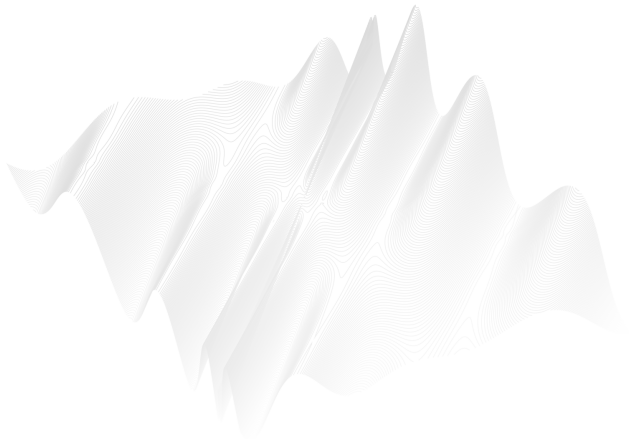
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### Macro only model

Yu and Moyeed (2001) show that the quantile specific estimation problem can be solved via the asymmetric Laplace distribution (ALD). Hence we can estimate the linear quantile function as:

$$Q(y_{i|x_i}) = x_i^T \beta(\tau) + \epsilon_{i\tau},$$

- ▶  $\beta(\tau)$  quantifies the quantile specific impact.
- ▶  $\epsilon_{i\tau} \sim AL(0, \sigma, \tau)$

### Random effects model

By including a quantile specific random effect, the location parameter changes to  $\mu = x_i^T \beta(\tau) + F_{t(i)}(\tau)$ .

Therefore, the linear conditional quantile function changes to:

$$Q(y_{i|x_i}) = x_i^T \beta(\tau) + F_{t(i)}(\tau) + \epsilon_{i\tau},$$

- ▶  $F_{t(i)}(\tau)$  is normally distributed and corresponds to the random effect and  $t(i)$  corresponds to the quarter of default  $t$

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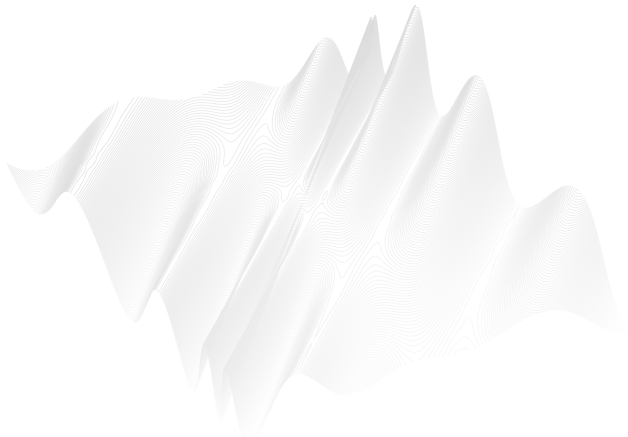
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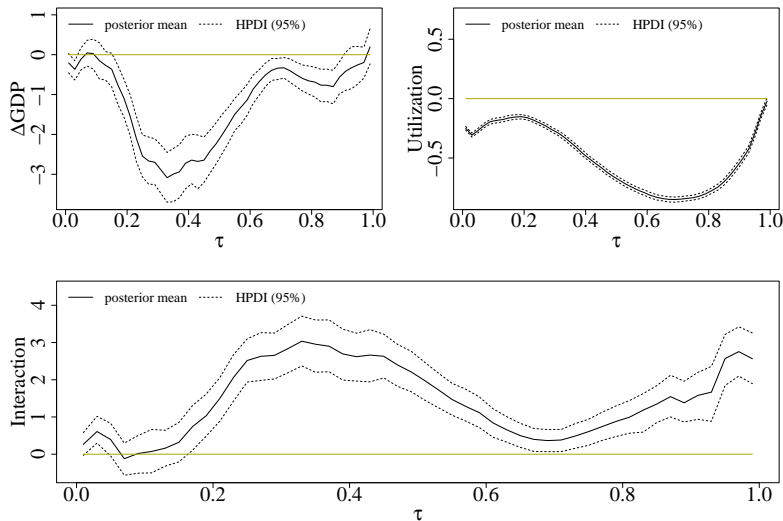
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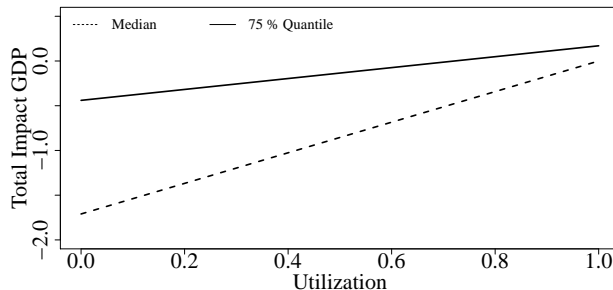


## Coefficient estimates — Macro variables and interaction in Europe



## Interaction term

**Total effect of the macro variable:**  $\beta_{GDP}^{(-)} + \beta_{Interaction}^{(+)} Utilization$

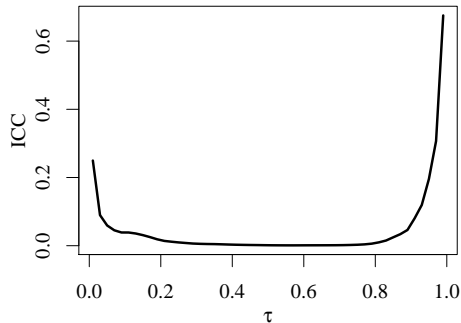
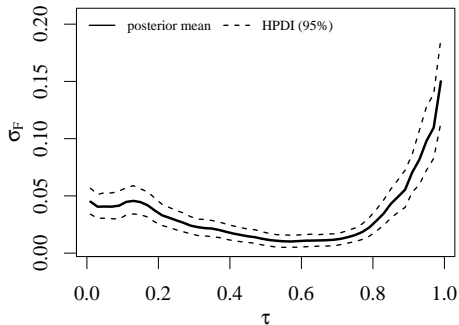


## Implications

- The economic environment impacts especially less drawn lines
- The effect of the macro variable is decreasing in quantiles

## Coefficient estimates — Random effects model

$$ICC = \frac{\sigma_F^2}{\sigma_F^2 + \sigma_\epsilon^2}$$



$$HMI = \frac{2}{n} \sum_{i=1}^n |p_{\text{empirical},i} - p_{\text{theoretical},i}|$$

The lower the calculated HMI, the better the distributional fit. A perfect fit results in an HMI of zero.

(a) USA

	Quantile Regression	OLS
<b>Mean</b>	0.0458	0.0823
<b>Standard deviation</b>	0.0080	0.0067

(b) Europe

	Quantile Regression	OLS
<b>Mean</b>	0.1216	0.1616
<b>Standard deviation</b>	0.0170	0.0130

Note: The table shows means, standard deviations of the HMI over the 1,000 sampled portfolios in each region. The HMI summarizes the absolute deviations from the perfect fit. Hence, the lower the value, the better the distributional fit. For the European Data set, the Random Effects Model is used, as it turned out to be superior. The random effects in the Random Effects Model are set to their expectation for prediction. The Macro Only Model is used in the US American data set.



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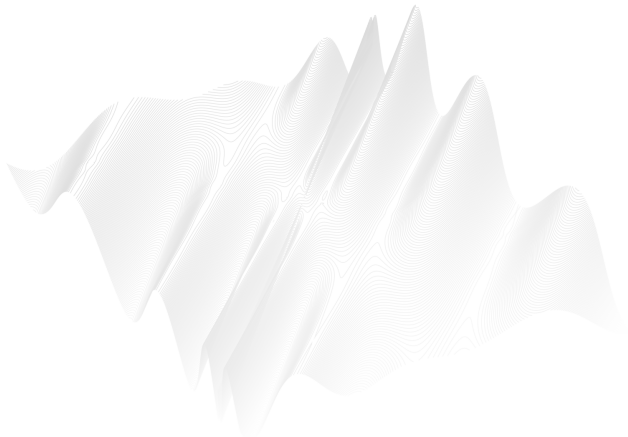
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## Main Findings

### 1. Drivers of credit line exposure

- We show that the economic environment is most important for less drawn lines (Interaction)
- Macroeconomic variables are not evident in the tails of the distribution
- We show that the impact of covariables varies strongly over quantiles

### 2. Differences in Regions

- The empirical results suggest differences between US American and European credit lines, especially over time

### 3. Downturn periods

- We show that, the linear model provides a rather poor distributional fit, especially in downturn periods.
- The macro variables cannot produce a sufficiently conservative distribution in the European sample
- Random effects incorporate the time varying nature of credit lines
- During economic downturns, a stressed random effect provides a more favourable fit

Thank you  
for your attention !

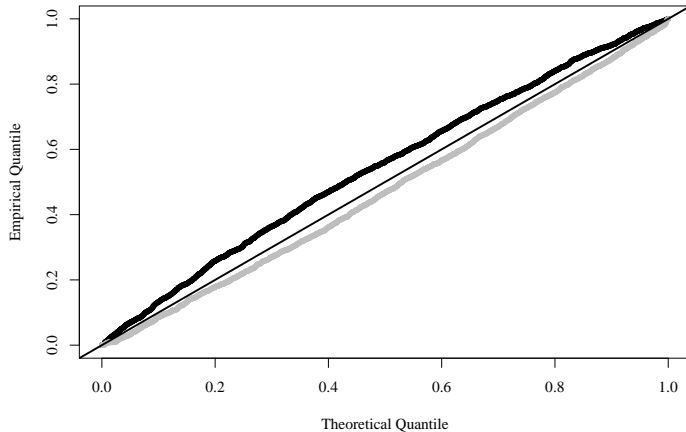


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# BACKUP

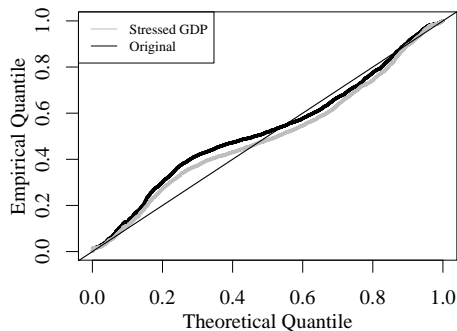
## Random effects model — Global Financial Crisis



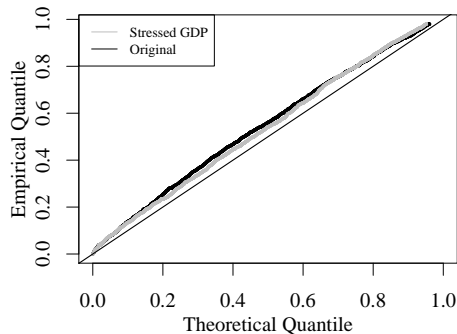
### Implications

→ Stressing the random effect results in a sufficiently conservative distribution

## Linear Model



## Bayesian Quantile Regression



# Macro only model

(a) USA

(b) Europe

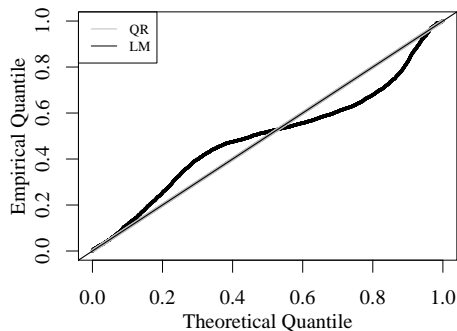
Variable	Level	$\tau = 0.05$	$\tau = 0.50$	$\tau = 0.95$	LM	$\tau = 0.05$	$\tau = 0.50$	$\tau = 0.95$	LM
<b>Intercept</b>		0.128 <sup>ooo</sup>	0.599 <sup>ooo</sup>	1.125 <sup>ooo</sup>	0.704 <sup>ooo</sup>	0.132 <sup>ooo</sup>	0.815 <sup>ooo</sup>	1.099 <sup>ooo</sup>	0.073 <sup>ooo</sup>
<b>Facility</b>	ST revolver	-0.042 <sup>ooo</sup>	-0.010 <sup>oo</sup>	-0.019 <sup>ooo</sup>	-0.042 <sup>ooo</sup>	0.017 <sup>o</sup>	0.015 <sup>oo</sup>	-0.013 <sup>oo</sup>	0.027
	Overdrafts					-0.029 <sup>ooo</sup>	0.012 <sup>ooo</sup>	0.220 <sup>ooo</sup>	0.045 <sup>ooo</sup>
<b>Industry</b> (FIRE)	Agriculture	-0.120 <sup>ooo</sup>	-0.008 <sup>o</sup>	0.018 <sup>o</sup>	-0.026	-0.013 <sup>o</sup>	0.004	0.117 <sup>ooo</sup>	0.044 <sup>o</sup>
	Mining	-0.115 <sup>ooo</sup>	-0.074 <sup>ooo</sup>	-0.024 <sup>oo</sup>	-0.087 <sup>ooo</sup>	0.029 <sup>o</sup>	0.007	0.611 <sup>ooo</sup>	0.110 <sup>o</sup>
	Constr.	-0.075 <sup>ooo</sup>	-0.017 <sup>oo</sup>	0.018 <sup>oo</sup>	-0.052 <sup>ooo</sup>	-0.050 <sup>ooo</sup>	-0.007 <sup>o</sup>	0.047 <sup>ooo</sup>	-0.001
	Manufact.	-0.072 <sup>ooo</sup>	-0.011 <sup>oo</sup>	0.115 <sup>ooo</sup>	-0.023 <sup>o</sup>	-0.053 <sup>ooo</sup>	-0.019 <sup>ooo</sup>	0.056 <sup>ooo</sup>	-0.014
	Transport.	-0.008	0.001	0.076 <sup>ooo</sup>	-0.009	-0.065 <sup>ooo</sup>	-0.021 <sup>ooo</sup>	0.037 <sup>oo</sup>	-0.001
	Wholesale	-0.088 <sup>ooo</sup>	-0.016 <sup>ooo</sup>	0.038 <sup>ooo</sup>	-0.046 <sup>ooo</sup>	-0.050 <sup>ooo</sup>	-0.020 <sup>ooo</sup>	0.019 <sup>o</sup>	-0.028 <sup>oo</sup>
	Service	-0.070 <sup>ooo</sup>	-0.010 <sup>oo</sup>	0.041 <sup>ooo</sup>	-0.027 <sup>oo</sup>	-0.043 <sup>ooo</sup>	-0.009 <sup>ooo</sup>	0.121 <sup>ooo</sup>	0.011
	Other	-0.070 <sup>ooo</sup>	-0.008 <sup>o</sup>	0.003	-0.05 <sup>ooo</sup>	-0.039 <sup>ooo</sup>	-0.027 <sup>oo</sup>	0.015 <sup>o</sup>	-0.054 <sup>ooo</sup>
<b>Seniority</b> (pari-passu)	Super senior	0.080 <sup>ooo</sup>	0.014 <sup>ooo</sup>	-0.098 <sup>ooo</sup>	-0.003	-0.040 <sup>ooo</sup>	0.001	0.045 <sup>ooo</sup>	-0.002
	Non senior	-0.037 <sup>ooo</sup>	0.008 <sup>o</sup>	-0.103 <sup>ooo</sup>	-0.034 <sup>oo</sup>	-0.045 <sup>ooo</sup>	0.060 <sup>ooo</sup>	0.461 <sup>ooo</sup>	0.143 <sup>ooo</sup>
	Unknown	0.133 <sup>ooo</sup>	0.025 <sup>ooo</sup>	-0.125 <sup>ooo</sup>	0.030				
<b>log(Limit)</b>		-0.016 <sup>ooo</sup>	-0.011 <sup>ooo</sup>	-0.007 <sup>ooo</sup>	-0.017 <sup>ooo</sup>	-0.013 <sup>ooo</sup>	-0.010 <sup>ooo</sup>	-0.037 <sup>ooo</sup>	-0.028 <sup>ooo</sup>
<b>Age</b>		-0.003 <sup>oo</sup>	-0.002 <sup>ooo</sup>	-0.004 <sup>ooo</sup>	-0.006 <sup>ooo</sup>	-0.002 <sup>ooo</sup>	0.000	0.004 <sup>ooo</sup>	0.000
<b>ΔGDP</b>		-0.129	-2.922 <sup>ooo</sup>	-0.207	-1.100 <sup>ooo</sup>	-0.114 <sup>ooo</sup>	-1.997 <sup>ooo</sup>	-0.255 <sup>o</sup>	-0.869 <sup>ooo</sup>
<b>Utilization</b>		-0.234 <sup>ooo</sup>	-0.480 <sup>ooo</sup>	-0.870 <sup>ooo</sup>	-0.475 <sup>ooo</sup>	-0.269 <sup>ooo</sup>	-0.687 <sup>ooo</sup>	-0.295 <sup>ooo</sup>	-0.382 <sup>ooo</sup>
<b>Interaction</b>		0.179	2.902 <sup>ooo</sup>	-0.398 <sup>o</sup>	1.076 <sup>o</sup>	0.393 <sup>oo</sup>	1.978 <sup>ooo</sup>	2.569 <sup>ooo</sup>	1.088 <sup>ooo</sup>

Note: This table shows the estimated posterior means for several selected quantiles. The first column inherits the name of the different independent variables. If they are categorical, the reference group is indicated in brackets. The second column illustrates the different levels of categorical variables. Statistical evidence is indicated by the following circles :<sup>o</sup> corresponds to substantial evidence (Odds  $\geq 3.2$ ), <sup>oo</sup> corresponds to strong evidence (Odds  $\geq 10$ ), <sup>ooo</sup> corresponds to decisive evidence (Odds  $\geq 100$ ).

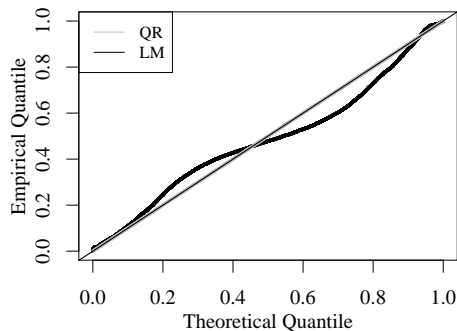


## Macro only model — In Sample fit

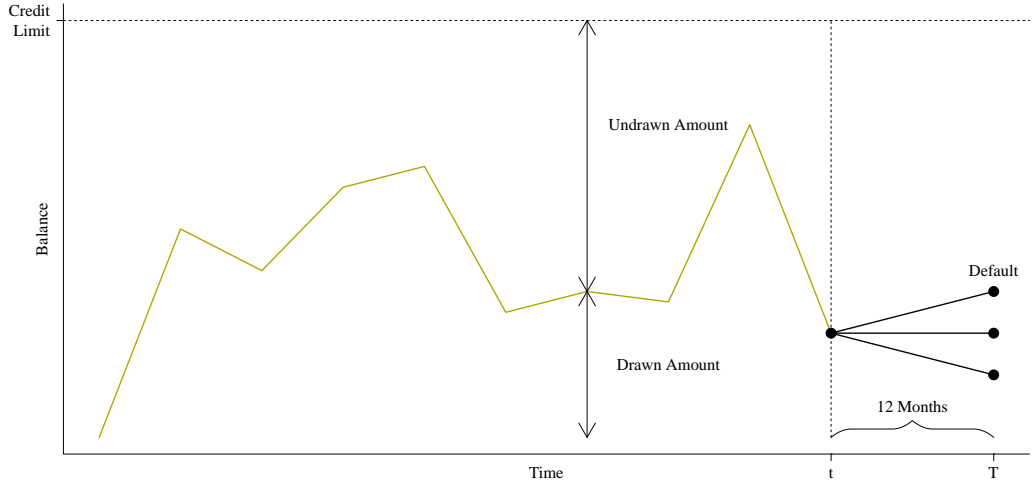
**USA**



**Europe**



# Exposure of credit lines



# Random effects model

Variable	Level	$\tau = 0.05$		$\tau = 0.50$		$\tau = 0.95$	
		Macro only	Random effects	Macro only	Random effects	Macro only	Random effects
<b>Intercept</b>		0.132 <sup>°°°</sup>	0.136 <sup>°°°</sup>	0.815 <sup>°°°</sup>	0.818 <sup>°°°</sup>	1.099 <sup>°°°</sup>	1.094 <sup>°°°</sup>
<b>Facility Type</b> (medium term)	Short term	0.017 <sup>°</sup>	0.005 <sup>°</sup>	0.015 <sup>°°</sup>	0.014 <sup>°°</sup>	-0.013 <sup>°°°</sup>	-0.015 <sup>°°°</sup>
	Overdraft	-0.029 <sup>°°°</sup>	-0.035 <sup>°°°</sup>	0.012 <sup>°°°</sup>	0.008 <sup>°°°</sup>	0.220 <sup>°°°</sup>	0.189 <sup>°°°</sup>
<b>Industry</b> (FIRE)	Agriculture	-0.013 <sup>°</sup>	0.006	0.004	0.007	0.117 <sup>°°°</sup>	0.148 <sup>°°°</sup>
	Mining	0.029 <sup>°</sup>	0.043 <sup>°°</sup>	0.007	0.004 <sup>°</sup>	0.611 <sup>°°°</sup>	0.543 <sup>°°°</sup>
	Construction	-0.050 <sup>°°°</sup>	-0.051 <sup>°°°</sup>	-0.007 <sup>°</sup>	-0.008 <sup>°</sup>	0.047 <sup>°°°</sup>	0.071 <sup>°°°</sup>
	Manufact.	-0.053 <sup>°°°</sup>	-0.048 <sup>°°°</sup>	-0.019 <sup>°°°</sup>	-0.021 <sup>°°°</sup>	0.056 <sup>°°°</sup>	0.073 <sup>°°°</sup>
	Transport	-0.065 <sup>°°°</sup>	-0.074 <sup>°°°</sup>	-0.021 <sup>°°°</sup>	-0.020 <sup>°°°</sup>	0.037 <sup>°°°</sup>	0.067 <sup>°°°</sup>
	Wholesale	-0.050 <sup>°°°</sup>	-0.046 <sup>°°°</sup>	-0.020 <sup>°°°</sup>	-0.021 <sup>°°°</sup>	0.019 <sup>°</sup>	0.041 <sup>°°°</sup>
	Service	-0.043 <sup>°°°</sup>	-0.034 <sup>°°°</sup>	-0.009 <sup>°°</sup>	-0.008 <sup>°</sup>	0.121 <sup>°°°</sup>	0.160 <sup>°°°</sup>
	Other	-0.039 <sup>°°°</sup>	-0.043 <sup>°°°</sup>	-0.027 <sup>°°°</sup>	-0.031 <sup>°°°</sup>	0.015 <sup>°</sup>	-0.010
<b>Seniority</b> (pari-passu)	Super senior	-0.040 <sup>°°°</sup>	-0.056 <sup>°°°</sup>	0.001	-0.003	0.045 <sup>°°°</sup>	0.015 <sup>°°°</sup>
	Non senior	-0.045 <sup>°°°</sup>	-0.052 <sup>°°°</sup>	0.060 <sup>°°°</sup>	0.058 <sup>°°°</sup>	0.461 <sup>°°°</sup>	0.371 <sup>°°°</sup>
<b>log(Limit)</b>		-0.013 <sup>°°°</sup>	-0.013 <sup>°°°</sup>	-0.010 <sup>°°°</sup>	-0.011 <sup>°°°</sup>	-0.037 <sup>°°°</sup>	-0.037 <sup>°°°</sup>
<b>Age</b>		-0.002 <sup>°°°</sup>	-0.002 <sup>°°°</sup>	0.000	0.000	0.004 <sup>°°°</sup>	0.006 <sup>°°°</sup>
<b>ΔGDP</b>		-0.114 <sup>°</sup>	0.045	-1.997 <sup>°°°</sup>	-1.952 <sup>°°°</sup>	-0.255 <sup>°</sup>	0.319
<b>Utilization</b>		-0.269 <sup>°°°</sup>	-0.261 <sup>°°°</sup>	-0.687 <sup>°°°</sup>	-0.677 <sup>°°°</sup>	-0.295 <sup>°°°</sup>	-0.287 <sup>°°°</sup>
<b>Interaction</b>		0.393 <sup>°°</sup>	0.483 <sup>°</sup>	1.978 <sup>°°°</sup>	1.960 <sup>°°°</sup>	2.569 <sup>°°°</sup>	2.221 <sup>°°°</sup>
<b><math>\sigma_F</math></b>			0.041 <sup>°°°</sup>		0.011 <sup>°°°</sup>		0.098 <sup>°°°</sup>

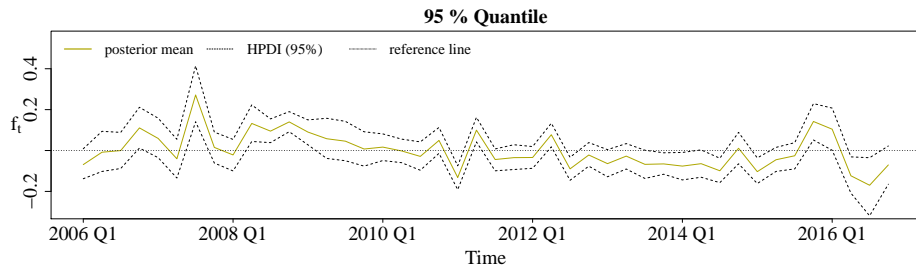
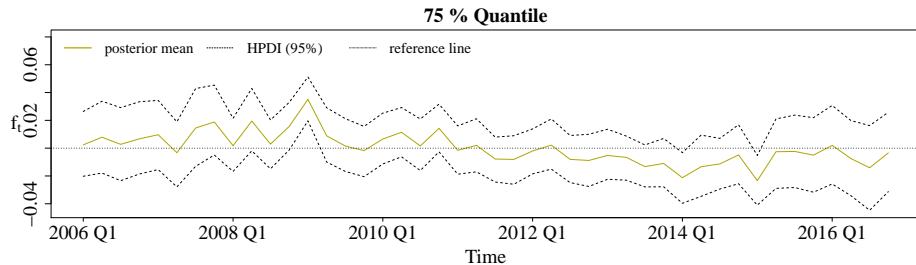
Note:

° corresponds to substantial evidence (Odds  $\geq 3.2$ )

°° corresponds to strong evidence (Odds  $\geq 10$ )

°°° corresponds to decisive evidence (Odds  $\geq 100$ )

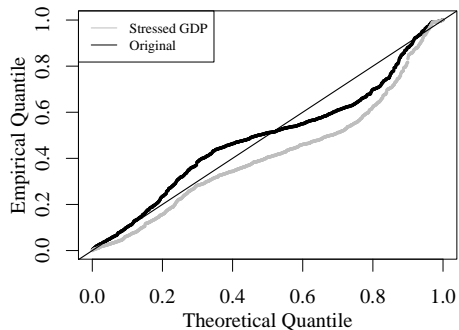
## Realization of the random effects



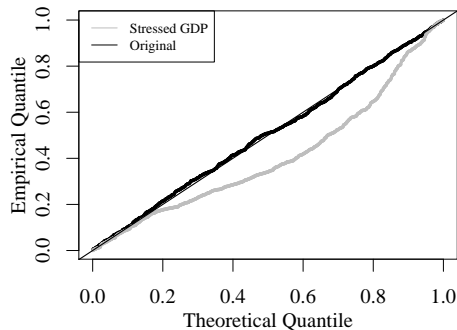
# Descriptive Statistics — USA

Variable	Level	Quantiles					Mean	STD	Obs.
		0.05	0.25	0.5	0.75	0.95			
<b>AUF</b>		-0.33	-0.07	0.00	0.03	0.59	0.03	0.26	3975
<b>log(Limit)</b>		9.69	11.63	12.96	14.51	16.72	12.86	2.92	3975
<b>Age</b>		0.10	0.84	1.99	3.79	7.92	2.73	2.63	3975
<b>Utilization</b>		0.15	0.80	1.00	1.00	1.00	0.84	0.28	3975
<b>ΔGDP</b>		-0.04	0.00	0.02	0.02	0.03	0.01	0.02	3975
<b>Facility type</b>	Medium term revolver	-0.31	-0.06	0.00	0.04	0.61	0.04	0.26	3258
	Short term revolver	-0.37	-0.11	0.00	0.00	0.45	-0.01	0.24	717
<b>Seniority</b>	Pari-passu	-0.40	-0.20	-0.02	0.00	0.61	-0.03	0.28	1006
	Super senior	-0.28	-0.05	0.00	0.06	0.54	0.04	0.25	1517
	Non senior	-0.36	-0.10	-0.02	0.01	0.77	0.03	0.31	147
	Unknown	-0.23	-0.03	0.00	0.06	0.59	0.06	0.25	1305
<b>Industry</b>	Finance, insurance, real estate (FIRE)	-0.28	-0.05	-0.02	0.00	0.42	-0.01	0.21	702
	Agriculture, forestry, fishing (AFF)	-0.34	-0.12	-0.01	0.02	0.38	-0.01	0.24	133
	Mining (MIN)	-0.39	-0.15	0.00	0.21	0.80	0.06	0.34	110
	Construction (CON)	-0.38	-0.05	0.00	0.11	0.55	0.04	0.27	426
	Manufacturing (MAN)	-0.34	-0.10	0.00	0.10	0.75	0.06	0.31	526
	Transp., commu., sanitary serv. (TCEGS)	-0.26	-0.04	0.00	0.06	0.75	0.06	0.28	223
	Wholesale and retail trade (WRT)	-0.35	-0.09	0.00	0.05	0.60	0.02	0.27	536
	Services (SER)	-0.32	-0.05	0.00	0.05	0.61	0.04	0.27	841
	Other (OTH)	-0.30	-0.08	0.00	0.00	0.30	-0.01	0.19	478

## Linear Model



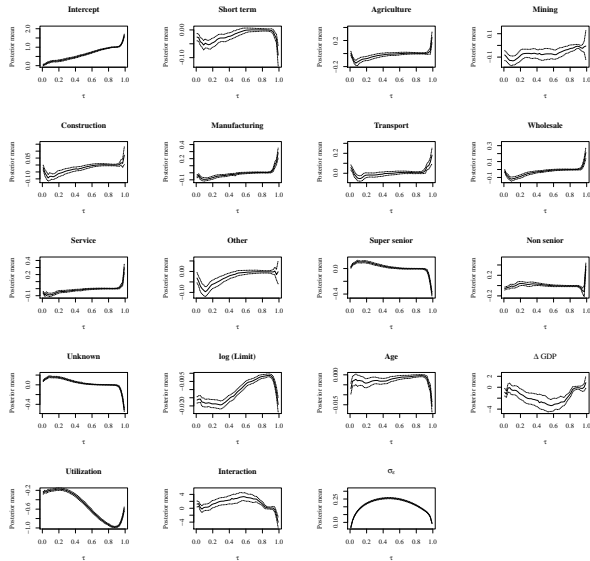
## Bayesian Quantile Regression



## Descriptive Statistics — Europe

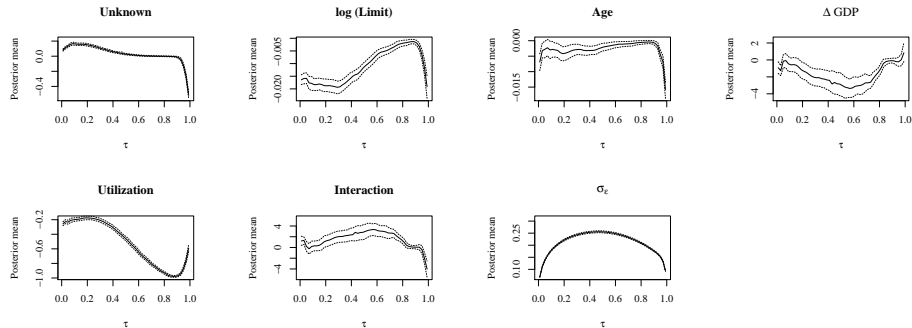
Variable	Level	Quantiles					Mean	STD	Obs.
		0.05	0.25	0.5	0.75	0.95			
<b>AUF</b>		-0.31	-0.01	0.03	0.36	1.04	0.20	0.41	10856
<b>log(Limit)</b>		4.21	8.33	10.71	12.58	15.18	9.55	4.64	10856
<b>Age</b>		0.00	1.21	3.43	6.48	19.26	5.25	6.80	10856
<b>Utilization</b>		0.00	0.48	0.97	1.00	1.00	0.72	0.39	10856
<b>ΔGDP</b>		-0.05	-0.01	0.01	0.02	0.03	0.00	0.02	10856
<b>Facility type</b>	Medium term revolver	-0.32	-0.03	0.01	0.18	0.86	0.11	0.32	3169
	Short term revolver	-0.29	0.00	0.00	0.06	0.95	0.09	0.31	375
	Overdraft	-0.31	0.00	0.06	0.50	1.12	0.25	0.44	7312
<b>Seniority</b>	Pari-passu	-0.29	0.00	0.04	0.38	1.06	0.21	0.41	9789
	Super senior	-0.39	-0.08	0.00	0.17	0.94	0.08	0.34	967
	Non senior	-0.40	-0.16	0.04	0.46	0.88	0.14	0.41	100
<b>Industry</b>	Finance, insurance, real estate (FIRE)	-0.28	-0.01	0.01	0.26	1.03	0.18	0.40	2722
	Agriculture, forestry, fishing (AFF)	-0.29	-0.01	0.05	0.36	1.00	0.20	0.38	427
	Mining (MIN)	-0.20	-0.02	0.04	0.40	1.13	0.24	0.45	39
	Construction (CON)	-0.31	0.00	0.08	0.59	1.19	0.27	0.46	1135
	Manufacturing (MAN)	-0.36	-0.02	0.02	0.39	1.05	0.20	0.42	1059
	Transp., commu., sanitary serv. (TCEGS)	-0.34	-0.04	0.02	0.35	1.01	0.18	0.41	496
	Wholesale and retail trade (WRT)	-0.34	-0.03	0.05	0.37	1.06	0.20	0.42	2051
	Services (SER)	-0.31	0.00	0.13	0.62	1.18	0.30	0.46	974
	Other (OTH)	-0.24	0.00	0.03	0.22	0.96	0.15	0.34	1953

# Coefficients I — USA — Macro only model

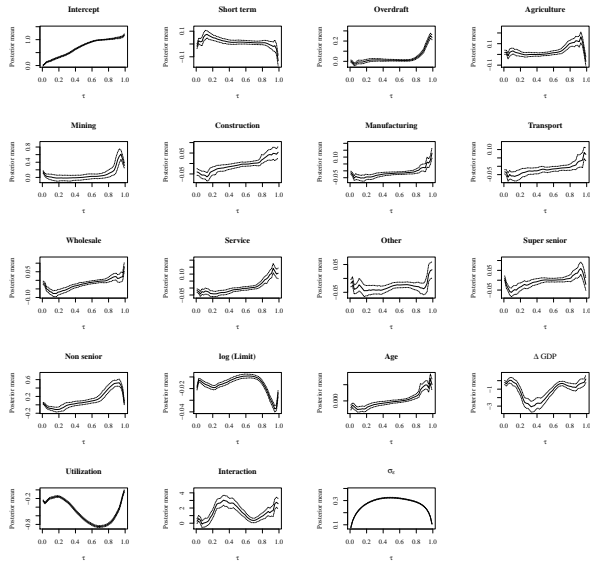




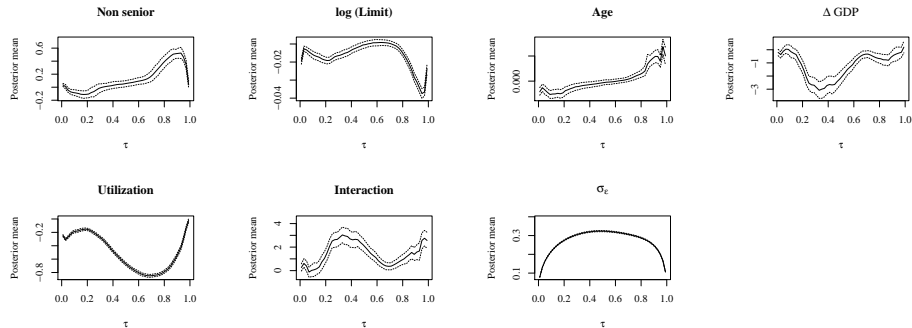
## Coefficients II — USA — Macro only model



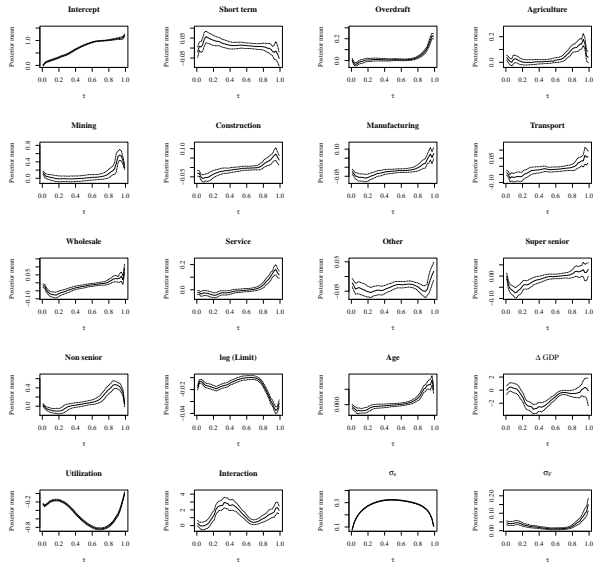
# Coefficients I — Europe — Macro only model



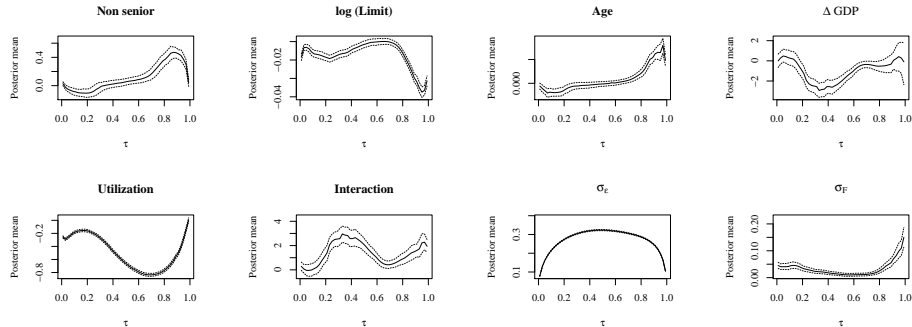
## Coefficients II — Europe — Macro only model



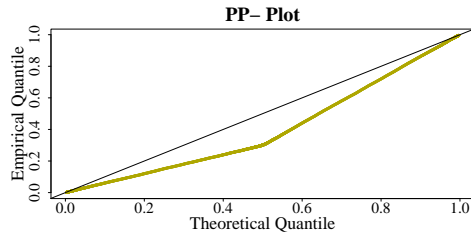
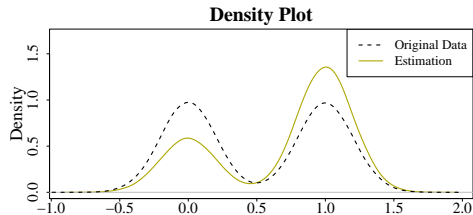
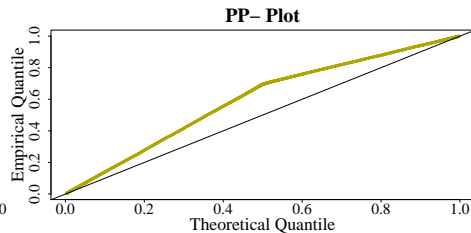
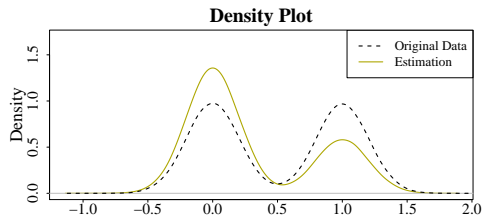
# Coefficients I — Europe — Random effects model



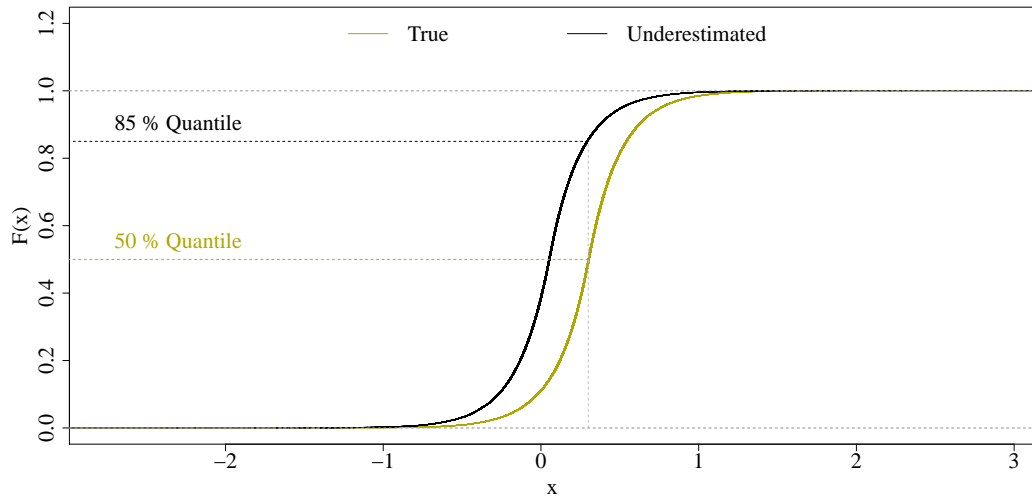
## Coefficients II — Europe — Random effects model



# Probability-Probability-Plot — Intuition



## Probability-Probability- Plot — Interpretation



### Bayesian Quantile Regression with random effects

Following Yu and Stander (2007) and Luo et al. (2012) a random variable of the asymmetric Laplace distribution can be formulated as a mixture representation of a standard normal and exponential random variable.

The linear quantile function changes to:

$$y = x_i^T \beta(\tau) + F_{t(i)}(\tau) + c_1 e + \sqrt{c_2 \sigma} e z,$$

where  $c_1 = \frac{1-2\tau}{\tau(1-\tau)}$ ,  $c_2 = \frac{2}{\tau(1-\tau)}$ ,  $z \sim N(0, 1)$  and  $e \sim \exp(\frac{1}{\sigma})$ .

The hierarchical Bayesian quantile regression can be formulated as follows:

$$f(y_i | \beta(\tau), F_{t(i)}(\tau), \sigma; e) = (2\pi c_2 \sigma e)^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2\pi c_2 e} (y_i - x_i^T \beta(\tau) - F_{t(i)}(\tau) - c_1 e)^2 \right\}$$
$$y = x_i^T \beta(\tau) + F_{t(i)}(\tau) + c_1 e + \sqrt{c_2 \sigma} e z$$



$$F_{t(i)}(\tau) \sim N(0, \sigma^F(\tau))$$

$$\sigma^F(\tau) \sim N(0, 1e+5)[0, \infty]$$

$$\beta(\tau) \sim N(0, 1e+5)$$

$$\sigma \sim N(0, 1e+5)[0, \infty]$$